Permeable values and energetic sets in BCK/BCI-algebras

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Abstract

The notions of $(\in, \in\vee q)$-permeable S-value and $(\in, \in\vee q)$-permeable I-value are introduced, and related properties are investigated. Relations among $(\in, \in\vee q)$-fuzzy subalgebra, $(\in, \in\vee q)$-fuzzy ideal, (strong) lower and (strong) upper level sets, $(\in, \in\vee q)$-permeable S-value, $(\in, \in\vee q)$-permeable I-value, S-energetic set, I-energetic set, right stable set and right vanished set are discussed.

1 Definitions

▶ **A fuzzy set μ in a BCK/BCI-algebra X is called a fuzzy subalgebra of X if it satisfies:**

$$(\forall x, y \in X)\,(\mu(x * y) \geq \min\{\mu(x), \mu(y)\})\,. \tag{1.1}$$

1 Definitions

▶ **A fuzzy set μ in a BCK/BCI-algebra X is called an anti fuzzy subalgebra (see [1]) [1] of X if it satisfies:**

$$(\forall x, y \in X)\,(\mu(x * y) \leq \max\{\mu(x), \mu(y)\})\,. \tag{1.2}$$

[1] S. M. Hong and Y. B. Jun, **Anti fuzzy ideals in BCK-algebras,** Kyungpook Math. J. **38** (1998), 145–150.

1 Definitions

▶ **A fuzzy set μ in a BCK/BCI-algebra X is called a fuzzy ideal of X if it satisfies:**

$$(\forall x \in X)\,(\mu(0) \geq \mu(x))\,. \tag{1.3}$$

$$(\forall x, y \in X)\,(\mu(x) \geq \min\{\mu(x * y), \mu(y)\})\,. \tag{1.4}$$

1 Definitions

► **A fuzzy set μ in a BCK/BCI-algebra X is called an anti fuzzy ideal of X (see [1])** [2] **if it satisfies:**

$$(\forall x \in X)\,(\mu(0) \leq \mu(x))\,. \tag{1.5}$$

$$(\forall x, y \in X)\,(\mu(x) \leq \max\{\mu(x * y), \mu(y)\})\,. \tag{1.6}$$

[2] S. M. Hong and Y. B. Jun, **Anti fuzzy ideals in BCK-algebras,** Kyungpook Math. J. **38** (1998), 145–150.

1 Definitions

A fuzzy set μ in a set X of the form

$$\mu(y) := \begin{cases} t \in (0,1] & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is said to be a fuzzy point[3] with support x and value t and is denoted by x_t.

[3] P. M. Pu and Y. M. Liu, **Fuzzy topology I, Neighborhood structure of a fuzzy point and Moore-Smith convergence,** J. Math. Anal. Appl. **76** (1980), 571–599.

1 Definitions

♣ **To say that $x_t \in \mu$ (resp. $x_t\, q\, \mu$) means that $\mu(x) \geq t$ (resp. $\mu(x)+t > 1$), and in this case, x_t is said to belong to (resp. be quasi-coincident with) a fuzzy set μ.**

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♣ **To say that $x_t \in\!\vee\, q \, \mu$ means that $x_t \in \mu$ or $x_t \, q \, \mu$.**

1 Definitions

♣ **A fuzzy set μ in a BCK/BCI-algebra X is called an $(\in, \in\vee q)$-fuzzy subalgebra (see [4])[4] of X if it satisfies:**

$$(\forall x, y \in X)(\forall t_1, t_2 \in (0,1]) \\ \left(x_{t_1} \in \mu,\ y_{t_2} \in \mu \ \Rightarrow (x * y)_{\min\{t_1,t_2\}} \in\vee\, q\, \mu\right). \tag{1.7}$$

[4]Y. B. Jun, **On (α, β)-fuzzy subalgebras of BCK/BCI-algebras,** Bull. Korean Math. Soc. **42** (2005), 703–711.

1 Definitions

♣ **A fuzzy set μ in a BCK/BCI-algebra X is called an $(\in, \in\vee q)$-fuzzy ideal (see [3])[5] of X if it satisfies:**

$$(\forall x \in X)(\forall t \in (0,1])\,(x_t \in \mu \ \Rightarrow\ 0_t \in\vee\, q\, \mu)\,, \tag{1.8}$$

$$\begin{aligned}&(\forall x, y \in X)(\forall t_1, t_2 \in (0,1])\\ &\big((x*y)_{t_1} \in \mu,\ y_{t_2} \in \mu \ \Rightarrow x_{\min\{t_1,t_2\}} \in\vee\, q\, \mu\big)\,.\end{aligned} \tag{1.9}$$

[5] Y. B. Jun, **On (α,β)-fuzzy ideals of BCK/BCI-algebras,** Sci. Math. Jpn. **60** (2004), 613–617.

2 $(\in, \in\vee q)$-permeable values

♣ **For a fuzzy set μ in X and $t \in [0,1]$, the upper and lower t-level sets of μ are defined as follows:**

$$U(\mu;t) := \{x \in X \mid \mu(x) \geq t\},\ L(\mu;t) := \{x \in X \mid \mu(x) \leq t\},$$

2 $(\in, \in\vee q)$-permeable values

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♣ **and the strong upper and strong lower t-level sets of μ are defined as follows:**

$$U^*(\mu;t) := \{x \in X \mid \mu(x) > t\},\ L^*(\mu;t) := \{x \in X \mid \mu(x) < t\}.$$

♣ **A non-empty subset A of a BCK/BCI-algebra X is said to be S-energetic (see [6])[6] if it satisfies:**

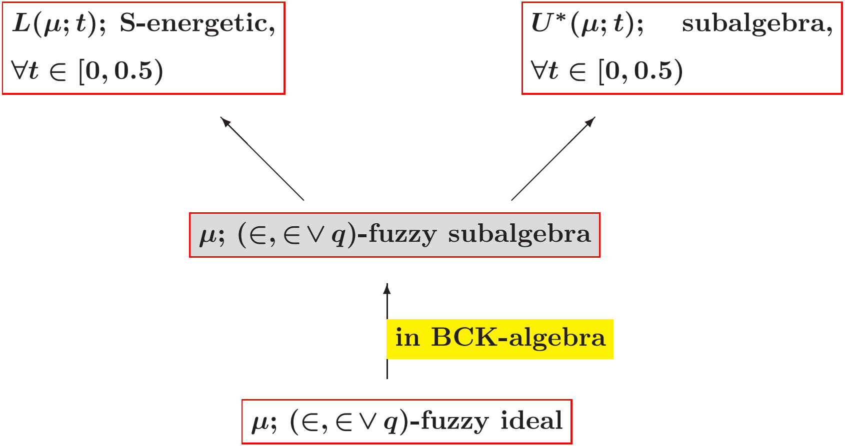
$$(\forall x, y \in X)\,(x * y \in A \;\Rightarrow\; \{x, y\} \cap A \neq \emptyset)\,. \tag{2.1}$$

[6] Y. B. Jun, S. S. Ahn and E. H. Roh, **Energetic subsets and permeable values with applications in BCK/BCI-algebras,** Appl. Math. Sci. **7** (2013), no. 89, 4425–4438.

♣ **A non-empty subset A of a BCK/BCI-algebra X is said to be I-energetic (see [6])[7] if it satisfies:**

$$(\forall x, y \in X)\,(y \in A \;\Rightarrow\; \{x, y * x\} \cap A \neq \emptyset)\,. \tag{2.2}$$

[7] Y. B. Jun, S. S. Ahn and E. H. Roh, **Energetic subsets and permeable values with applications in BCK/BCI-algebras,** Appl. Math. Sci. **7** (2013), no. 89, 4425–4438.


$L(\mu; t)$; S-energetic, $\forall t \in [0, 0.5)$
$U^*(\mu; t)$; subalgebra, $\forall t \in [0, 0.5)$
μ; $(\in, \in \vee q)$-fuzzy subalgebra
in BCK-algebra
μ; $(\in, \in \vee q)$-fuzzy ideal

Given $t \in \Lambda \subseteq [0,1]$,
$L^*(\mu; t)$; subalgebra

Given $t \in \Lambda \subseteq [0,1]$,
$U(\mu; t)$; S-energetic

$t \in \Lambda \subseteq [0,1]$; pearmeable S-value

μ; anti-fuzzy subalgebra

μ; anti-fuzzy ideal

in BCK-algebra

Given $t \in \Lambda \subseteq [0,1]$, $L^*(\mu;t)$; ideal

Given $t \in \Lambda \subseteq [0,1]$, $U(\mu;t)$; I-energetic

↓

$t \in \Lambda \subseteq [0,1]$; pearmeable I-value

↓ $(\forall x \in X)(\mu(0) \leq \mu(x))$

$t \in \Lambda \subseteq [0,1]$; pearmeable S-value

Definition

♣ **Let μ be a fuzzy set in a BCK/BCI-algebra X and $t \in \Lambda \subseteq [0,1]$. Then t is called an $(\in, \in \vee\, q)$-permeable S-value for μ if the following assertion is valid:**

$$(\forall x, y \in X)\,(x * y \in U(\mu; t) \;\Rightarrow\; \max\{\mu(x), \mu(y), 0.5\} \geq t)\,. \tag{2.3}$$

Example

♣ **Consider a BCK-algebra $X = \{0, 1, 2, 3, 4\}$ with the binary operation $*$ which is given in Table 1 (see [7]).**

Table 1: Cayley table for the binary operation "$*$"

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	1	0	0	0
3	3	1	1	0	0
4	4	3	3	1	0

Let μ be a fuzzy subset in X defined by

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.4 & 0.3 & 0.7 & 0.5 & 0.6 \end{pmatrix}.$$

It is routine to verify that $t \in [0.4, 0.7]$ is an $(\in, \in \vee q)$-permeable S-value for μ.

$t \in (0.5, 1]$; $(\in, \in\vee q)$-pearmeable S-value

(1) $U(\mu; t)$; S-energetic,

(2) $L^*(\mu; t)$; subalgebra

$t \in (0.5, 1]$; pearmeable S-value

μ; anti-fuzzy subalgebra

$t \in \Lambda \subseteq [0,1]$; $(\in, \in \vee\, q)$-pearmeable S-value

$t \in (0.5, 1]$

(1) $U(\mu;t)$; S-energetic,

(2) $L^*(\mu;t)$; subalgebra

$\mu(x) \geq \min\{\mu(x*y), \mu(y), 0.5\}$

μ; $(\in, \in\vee q)$-fuzzy ideal

$L(\mu; t)$; I-energetic, $\forall t \in [0, 0.5)$

$U^*(\mu; t)$; ideal, $\forall t \in [0, 0.5)]$

$\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$

μ; **anti-fuzzy ideal**

$t \in \Lambda \subseteq [0, 1]$; $(\in, \in \vee q)$-**permeable I-value**

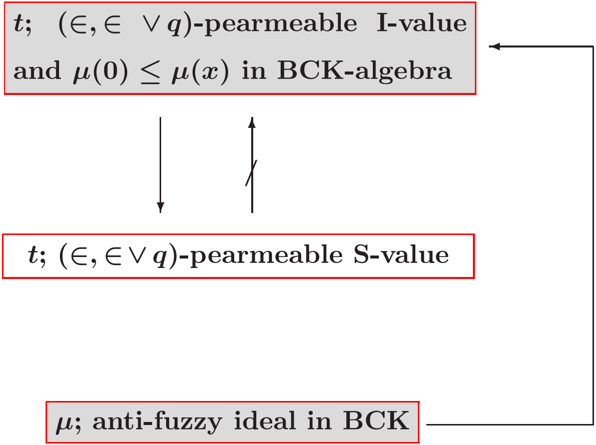
$\Lambda = (0.5, 1]$

$U(\mu; t)$; **I-energetic**, $\forall t \in (0.5.1]$

$t \in (0.5, 1]$; $(\in, \in \vee q)$-permeable I-value and $0 \notin U(\mu; t)$

$\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$ and $0 \notin U(\mu; t)$

$L^*(\mu; t)$; ideal, $\forall t \in (0.5.1]$


t; $(\in, \in \vee q)$-pearmeable I-value and $\mu(0) \leq \mu(x)$ in BCK-algebra
t; $(\in, \in \vee q)$-pearmeable S-value
μ; anti-fuzzy ideal in BCK

Definition

♣ **A nonempty subset A of a BCK/BCI-algebra X is said to be right vanished (see [6])[8] if it satisfies:**

$$(\forall x, y \in X)\,(x * y \in A \;\Rightarrow\; x \in A)\,. \tag{2.4}$$

♣ **A nonempty subset A of a BCK/BCI-algebra X is said to be right stable (see [6]) if $A * X := \{a * x \mid a \in A,\ x \in X\} \subseteq A$.**

[8] Y. B. Jun, S. S. Ahn and E. H. Roh, **Energetic subsets and permeable values with applications in BCK/BCI-algebras,** Appl. Math. Sci. **7** (2013), no. 89, 4425–4438.

$U(\mu;t)$ and $U^*(\mu;t)$ are right stable for $t \in (0, 0.5)$

↑

μ; $(\in, \in \vee q)$-fuzzy ideal in BCK

↓

$L(\mu;t)$ and $L^*(\mu;t)$ are right vanished for $t \in (0, 0.5)$

One of the best ways to learn math is to play with it.

Young Bae Jun (04 September 2009)

Thank you

for your Attention !!!

References

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[4] **Y. B. Jun, On (α, β)-fuzzy subalgebras of BCK/BCI-algebras, Bull. Korean Math. Soc. 42 (2005), 703–711.**

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[6] Y. B. Jun, S. S. Ahn and E. H. Roh, Energetic subsets and permeable values with applications in BCK/BCI-algebras, Appl. Math. Sci. 7 (2013), no. 89, 4425–4438.

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